Measurement of Analog Modulation Index with a Calibrated Radio Frequency Attenuator

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Abstract

This paper presents a method for the accurate and traceable measurement of the analog modulation index. A calibrated step attenuator was used as the main apparatus because it has a higher dynamic range and lower uncertainty than a spectrum analyzer or an oscilloscope. In amplitude modulation (AM), the modulation index is obtained from the amplitude difference between the carrier and the first sideband, as in the conventional method. The resolution and calibration uncertainties of the step attenuator were propagated to the measurement uncertainty of the modulation index. The uncertainty produced by the impedance mismatch and repeatability was also included. For frequency modulation (FM) and phase modulation (PM), the modulation index, β , was estimated (with the step attenuator) from the spectrum of each sideband through the nonlinear fitting of the Bessel function. Thus, the uncertainty of the fitting process was added to the uncertainty of the measurement. The three modulations, AM, FM, and PM, exhibited an expanded uncertainty (approximately 95% confidence level, $k = 2$) of 0.372% for 50% nominal depth of the AM, 88.8 Hz for the peak frequency deviation of 10 kHz, and 0.88 mrad for a 0.1 radian modulation index, respectively.

Key Words: Amplitude Modulation, Frequency Modulation, Measurement Uncertainty, Metrology, Microwave Measurement, Modulation Index, Phase Modulation, Sensitivity Analysis.

I. INTRODUCTION

Since the early 1900s, analog modulation has been used in wireless communication. It is still an important modulation scheme despite the extensive use of digital communications [1–3]. Analog modulation is one of the signal and pulse characteristics under the classification of the electricity and magnetism CMC (calibration and measurement capability) of the International Bureau of Weights and Measures. However, only a few National Measurement Institutes (NMIs) are listed. Moreover, there is no well-known measurement method with traceability.

Analog modulation can be measured in the frequency domain

or the time domain [4–6]. In the preliminary work, we found that measuring the modulation index in the frequency domain is more effective than measurement in the time domain to reduce uncertainty. Thus, we measured the analog modulation index in the frequency domain in this study. Measuring the absolute power of a signal is usually important. Measuring the modulation index, however, requires a signal strength relative to the carrier, not the absolute power on each sideband.

In frequency modulation (FM) or phase modulation (PM), the modulation index is usually measured with the Bessel null technique. This method uses the characteristics of carrier or sideband signals have a value of zero when the modulation in-

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dex achieves a specific value [5, 7]. Thus, the modulation index can only be measured at a specific value.

In this paper, we used a calibrated attenuator as the primary standard for the measurement of the modulation index. It is widely known that attenuators provide high accuracy and low uncertainty in the measurement of the relative signal strength. Moreover, the use of a calibrated attenuator readily makes the analog modulation index traceable to the measurement standard. In addition, we propose a method to measure the arbitrary FM and PM modulation indices through nonlinear fitting to overcome the limitation of the Bessel null technique. Finally, we analyze the measurement uncertainty of an analog modulation. To the best of our knowledge, this is the first comprehensive paper on analog modulation uncertainty analysis.

This paper is organized as follows. Section II describes the system for measuring the modulation index and the experimental evaluation of the effects of the impedance mismatch. Section III explains the measurement of the amplitude modulation (AM), FM, and PM modulation indices. Section IV explains the measurement of the relative amplitude of the sideband with the attenuator. Section V analyzes the uncertainty of each modulation index measurement, and Section VI concludes the paper.

II. MEASUREMENT SETUP

1. Measurement Setup

Fig. 1(a) shows a block diagram and a photograph of the proposed measurement system for the analog modulation indices. The modulated signal generated by the device under test (DUT) was passed through the calibrated attenuator to the spectrum analyzer. The spectrum analyzer measured the strength of the carrier and sideband signals. Note that the spectrum analyzer had not yet been calibrated. The DUT and the spectrum analyzer were synchronized with 10 MHz, and each device was remotely controlled via a general purpose interface bus (GPIB, IEEE 488). We used a step attenuator with a resolution of 0.1 dB, which was calibrated at 0 dB to 100 dB [8].

2. Impedance Mismatch

Prior to measuring, we analyzed the effect of the impedance mismatch. The state of the impedance matching was adjusted by the insertion of an impedance stub tuner between the spectrum analyzer and the step attenuator (Fig. 2). The analog modulation index was measured by taking the relative amplitude difference between the carrier and sidebands (see a detailed explanation in Section III). Table 1 shows the measurement results. The absolute power of the carrier varied greatly on the basis of the status of the impedance stub tuner. The relative difference between the first sideband and the carrier, however,

Fig. 1. (a) Block diagram of the measurement system for measuring analog modulation index. The calibrated attenuator is used to correct the raw data measured by the spectrum analyzer. (b) Calibration procedure. First, the modulation signal is measured using the spectrum analyzer. Then, the attenuator is adjusted until the carrier signal is equal to the sideband signal.

Fig. 2. Measurement setup for the impedance mismatch effect between DUT and the measurement system. The impedance stub tuner is attached to the input port of the spectrum analyzer to produce arbitrary impedance matching status.

Table 1. Effect of the impedance mismatch

Status	Power of carrier (dBm)	Difference between carrier and first sideband (dB)
A	-10	-12.482
B	-18	-12.483
C	-28	-12.467
D	-47	-12.471

did not change regardless of the impedance matching status of the impedance tuner. Thus, the impedance mismatch did not significantly affect the analog modulation index measurement.

III. MODULATION INDEX MEASUREMENT

1. Amplitude Modulation

AM changes the amplitude of the carrier, $c(t)$, on the basis of the modulating signal or message, $s(t)$. If $s(t)$ is a continuous wave (CW), similar to that of the carrier, the AM signal, $y(t)$, can be easily expressed as follows:

$$c(t) = A \cos(2\pi f_c t + \phi_c) \quad (1)$$

$$s(t) = Am \cos(2\pi f_m t + \phi_m) \quad (2)$$

$$\begin{aligned} y(t) &= \left(1 + \frac{s(t)}{A}\right) c(t) \\ &= A \cos(2\pi f_c t + \phi_c) + \frac{1}{2} Am \left(\cos(2\pi(f_c + f_m)t + \phi_c + \phi_m) \right) \\ &\quad + \frac{1}{2} Am \left(\cos(2\pi(f_c - f_m)t + \phi_c - \phi_m) \right) \end{aligned} \quad (3)$$

where A is the amplitude of the carrier and m is the modulation index of AM. Thus, the amplitude of the sideband signal ($f_c \pm f_m$) changes in proportion to the modulation index, and the modulation index is calculated as follows:

$$m = \frac{2V_{SB}}{V_C} = 10^{\frac{\Delta P + 20 \log_{10}(2)}{20}} \quad (4)$$

$$\Delta P = 20 \log_{10}(V_{SB} / V_C) \quad (5)$$

where V_C and V_{SB} are the amplitude of the carrier and sideband, respectively. The differences between the carrier and sideband signals on the dB scale are represented by ΔP , and it is easily obtained from the delta marker on the spectrum analyzer. In this paper, all the measured values were corrected with the calibrated attenuator for traceability to the measurement standard (details are provided in Section IV).

Fig. 3 shows the AM measurement results. For the DUT, the modulation index and the frequency of the modulating signal, f_m , were set to 50% and 50 kHz, respectively. Thus, the frequency interval between the carrier and sideband signals became 50 kHz. The nonlinear distortion was generated in the DUT because the large modulation index of 50% produced additional harmonics at $f_c \pm nf_m$ ($n > 2$). In theory, both of the first sidebands should have the same amplitude; however, they should exhibit asymmetric characteristics in the presence of FM distortion [9]. In this study, the average value of the two signals was used as the V_{SB} because the difference between them was very small, less than 0.005 dB. This value is counted as the standard uncertainty for the unbalanced sidebands, assuming a rectangular distribution of ± 0.0025 dB.

2. Angle Modulation (FM and PM)

FM is expressed as (6), where Δf_p indicates the peak frequency deviation. Therefore, the changes in the frequency of $y(t)$

Fig. 3. Measurement results for amplitude modulation (AM) with 50% of the AM depth where the carrier frequency is 500 MHz and the modulation repetition rate is 50 kHz. The uncorrected (raw measurement) and corrected (using the calibrated attenuator) amplitudes are represented with '—O' and 'X', respectively.

depend on $s(t)$. The amplitude, A , is time invariant:

$$y(t) = A \cos\left(2\pi f_c t + 2\pi \Delta f_p \int_0^t s(\tau) d\tau\right) \quad (6)$$

If $s(t)$ is CW, as was the case for AM, (6) can be represented as follows:

$$\begin{aligned} y(t) &= A \cos\left(2\pi f_c t + \phi_c + \frac{\Delta f_p}{f_m} \sin(2\pi f_m t + \phi_m)\right) \\ &= A \cos(2\pi f_c t + \phi_c + \beta \sin(2\pi f_m t + \phi_m)) \end{aligned} \quad (7)$$

where $\beta = \Delta f_p / f_m$ indicates the FM modulation index. Usually, the peak frequency deviation, Δf_p , is measured for a given frequency of the modulating signal, f_m . Eq. (7) also represents PM given that $s(t) = \cos(2\pi f_m t)$ causes a phase change on $y(t)$. Thus, when $s(t)$ is a sinusoidal function, the FM and PM have the same form, and they are commonly referred to as angle modulation. Mathematically, (7) can be expressed as the sum of the Bessel function of the first kind J as follows:

$$\begin{aligned} A \left(\sum_{k=0}^{\infty} |J_k(\beta)| \cos(2\pi(f_c + kf_m)t + \phi_c + k\phi_m) \right. \\ \left. + \sum_{k=-\infty}^{-1} |J_k(\beta)| \cos(2\pi(f_c + kf_m)t + \phi_c + k(\phi_m + \pi)) \right) \end{aligned} \quad (8)$$

where k is the order of the sidebands. For example, when $s(t)$ is a CW, the amplitude values of the carrier, first sideband, and second sideband are $AJ_0(\beta)$, $AJ_1(\beta)$, and $AJ_2(\beta)$, respectively. As stated in previous section, the calculation of analog modulation index requires the relative difference between the carrier and each sideband. Therefore, the amplitude of carrier $AJ_0(\beta)$ can be normalized to 1.

Fig. 4. Measurement results for phase modulation (PM) where the carrier frequency (f_c) is 500 MHz and the modulating signal (f_m) frequency is 100 kHz. (a) $\beta = 0.1$; (b) $\beta = 3$. The calibrated amplitude using the attenuator and the fitting result are represented with ‘—O’ and ‘X’, respectively.

Then, the angle modulation index β can be estimated by minimizing the difference between the measurement and estimated values through nonlinear least squares fitting:

$$\sum_{k=-n}^n \frac{y_{\text{meas}}(k)}{y_{\text{meas}}(0)} - \frac{|J_k(\beta)|}{|J_0(\beta)|} \quad (9)$$

Here $y_{\text{meas}}(k)$ is the measured value of the k -th sideband. The *nlinfit* function in the MATLAB environment was applied as the estimator [10]. This is based on the Levenberg-Marquardt nonlinear least squares algorithm [11]. Nonlinear fitting problems often reach the local minima or slowly converge depending on the initial value. Robust results were obtained from the initial β which is Δf_p divided by f_m (both are the values for the DUT setting).

Fig. 4 shows the measurement results. As was previously described, the number and shape of the generated sidebands depend on β . The fitted results (“x” marks) on the basis (9) agree well with the measured results (solid line). Estimation of β using 23 sideband measurements takes less than 0.05 seconds on an Intel Xeon E5 2.4 GHz CPU.

IV. CALIBRATION OF SPECTRUM ANALYZER WITH THE ATTENUATOR

The power of the frequency spectrum can usually be measured with a spectrum analyzer. As described in the previous section, the relative difference of the sideband signals, rather than the absolute power spectrum, was necessary to assess the measurement modulation index. Thus, for reducing measurement uncertainty and increasing precision, the calibrated atten-

uator is superior to the spectrum analyzer because of its higher dynamic range and lower uncertainty.

The difference between the carrier and sidebands, ΔP , as measured by a spectrum analyzer, can be corrected with a calibrated attenuator, as shown in Fig. 1(b). First, the modulation signal is measured using the spectrum analyzer. Then, the attenuator is adjusted until the carrier signal is equal to the sideband signal. Now the setting value of the attenuator Att_{set} can replace the uncalibrated ΔP . However, the reduced carrier signal may not be exactly the same as the power level of sidebands. Thus, the calibrated value can be calculated as follows:

$$\Delta P_{\text{corr}} = \Delta P_{\text{SA}} - Att_{\text{set}} - \varepsilon \quad (10)$$

where ΔP_{corr} is the corrected difference on the basis of the attenuator. ΔP_{SA} is the raw measurement using the spectrum analyzer, and ε is the residual error that represents the power difference between the sidebands and the reduced carrier signal after the attenuator is set.

V. UNCERTAINTY ANALYSIS

1. Uncertainty of Amplitude Modulation Measurement

The measuring uncertainty of the AM modulation index is caused by the uncertainty introduced by the attenuator and the impedance mismatch between the DUT and the attenuator. The attenuator had a reading uncertainty of $0.1/(2\sqrt{3})$ dB obtained from a resolution of 0.1 dB and a calibration uncertainty of 0.012 dB. If we actually correct the impedance mismatch by using the measured impedance values, the associated uncertainty will significantly increase due to the uncertainty of a vector network analyzer. To resolve this, we measured the difference the carrier and first sideband of the AM signal under an arbitrary matching condition in Table 1. To account for the uncertainty of the impedance mismatch for the scheme above, the standard deviation was obtained from the measured values in Table 1. Next, (4) was differentiated to propagate each uncertainty to the AM modulation index. The obtained sensitivity coefficient was $\log(10) \times 10^{\frac{\Delta P + 20 \log 10(2)}{20}} / 20$. The coefficient was dependent on ΔP ; thus, the large modulation index produced a large measurement uncertainty. Type A uncertainty was evaluated from 10 repeated measurements. The uncertainty budget is summarized in Table 2. The modulation index of 50% was measured, and the combined standard uncertainty, $u_c(m)$ 0.186%, was obtained by taking the root sum of squares of the uncertainty contributions presented in Table 2. Since the effective degree of freedom is sufficiently large, the expanded uncertainty $U(m)$ at approximately 95% confidence level is obtained by multiplying $u_c(m)$ by 2.

2. Uncertainty for Frequency and Phase Modulation Measurement

Tables 3 and 4 show the uncertainty budget for FM and PM, respectively. In both cases, the angle modulation required additional uncertainty for estimating β from the measured amplitude of each sideband. Unfortunately, the fitting uncertainty for β could no longer be regarded as analytical. Instead, it was calculated from the residuals, which were calculated from the difference between the fitting model and measurements [12, 13]:

$$\sigma_{\text{residual}}^2 = \sum_{k=-n}^n \left| \frac{y_{\text{meas}}(k)}{y_{\text{meas}}(0)} - \frac{|J_k(\beta)|}{|J_0(\beta)|} \right|^2 / (l - p) \quad (11)$$

$$\sigma_{\beta}^2 = \left(\frac{\partial \sigma_{\text{residual}}}{\partial \beta} \right)^2 \sigma_{\text{residual}}^2 \quad (12)$$

where l is the number of sidebands used in the measurement, including the carrier; thus, l is equal to $2n + 1$. The number of unknown parameter β used in the fitting process, i.e., 1, is indicated by p . Therefore, the covariance for the fitting parameter

Table 2. Uncertainty budget for AM modulation ($f_m = 50$ kHz and $m = 50\%$)

Uncertainty	Standard uncertainty	Sensitivity coefficient	Uncertainty contribution	Type	Probability distribution	Degree of freedom
Attenuator						
Resolution	0.029 dB	$\log(10) \times 10^{\frac{\Delta P + 20 \log 10(2)}{20}} / 20$	0.167%	B	Rectangular	∞
Calibration uncertainty	0.012 dB	$\log(10) \times 10^{\frac{\Delta P + 20 \log 10(2)}{20}} / 20$	0.069%	B	Normal	∞
Impedance mismatch	0.008 dB	$\log(10) \times 10^{\frac{\Delta P + 20 \log 10(2)}{20}} / 20$	0.046%	B	Normal	∞
Imbalance of sidebands	0.002 dB	$\log(10) \times 10^{\frac{\Delta P + 20 \log 10(2)}{20}} / 20$	0.012%	B	Rectangular	∞
Repeatability	0.003%	1	0.003%	A	Normal	9
$u_c(m)$	-	-	0.186%	-	Normal	∞
$U(m)$	-	-	0.372%	-	Normal ($k = 2$)	-

Table 3. Uncertainty budget for FM modulation ($f_m = 100$ kHz and $\Delta f_p = 10$ kHz, 50 kHz, 100 kHz)

Uncertainty	Standard uncertainty			Sensitivity coefficient	Uncertainty contribution			Type	Probability distribution	Degree of freedom
	10 kHz	50 kHz	100 kHz		10 kHz	50 kHz	100 kHz			
Fitting uncertainty	1.6 Hz	21.1 Hz	10.5 Hz	1	1.6 Hz	21.1 Hz	10.5 Hz	B	Normal	-
Attenuator										
Resolution		0.029 dB			40.9 Hz	182.7 Hz	261.8 Hz	B	Rectangular	∞
Calibration uncertainty		0.012 dB			17.0 Hz	76.3 Hz	108.6 Hz	B	Normal	∞
Impedance mismatch	2.9 Hz	14.8 Hz	29.6 Hz	1	2.9 Hz	14.8 Hz	29.6 Hz	B	Normal	∞
Repeatability	1.4 Hz	11 Hz	17.7 Hz	1	1.4 Hz	11 Hz	17.7 Hz	A	Normal	9
$u_c(m)$		-		-	44.4 Hz	213.5 Hz	285.7 Hz	-	Normal	∞
$U(m)$		-		-	88.8 Hz	427.0 Hz	571.4 Hz	-	Normal ($k = 2$)	-

Table 4. Uncertainty budget for PM modulation ($f_m = 100$ kHz and $\beta = 0.1$ rad, 1 rad, 3 rad)

Uncertainty	Standard uncertainty			Sensitivity coefficient	Uncertainty contribution			Type	Probability distribution	Degree of freedom
	0.1 rad	1 rad	3 rad		0.1 rad	1 rad	3 rad			
Fitting uncertainty	0.03 mrad	0.34 mrad	0.11 mrad	1	0.03 mrad	0.26 mrad	0.11 mrad	B	-	-
Attenuator										
Resolution		0.029 dB		-	0.40 mrad	2.63 mrad	1.43 mrad	B	Rectangular	∞
Calibration uncertainty		0.012 dB		-	0.17 mrad	1.09 mrad	0.60 mrad	B	Normal	∞
Impedance mismatch	0.07 mrad	0.21 mrad	0.21 mrad	1	0.07 mrad	0.21 mrad	0.21 mrad	B	Normal	∞
Repeatability	0.03 mrad	0.15 mrad	0.15 mrad	1	0.03 mrad	0.15 mrad	0.15 mrad	A	Normal	9
$u_c(m)$		-		-	0.44 mrad	2.85 mrad	1.56 mrad	-	Normal	∞
$U(m)$		-		-	0.88 mrad	5.70 mrad	3.12 mrad	-	Normal ($k = 2$)	-

σ_β^2 was calculated using (12) with the fitting residuals $\sigma_{\text{residual}}^2$. It must be noted that σ_β directly became the fitting uncertainty of β in PM. In the case of FM, the sensitivity coefficient was f_m because the peak frequency deviation was $\Delta f_p = f_m \beta$. Thus, the covariance of the peak frequency deviation $\sigma_{\Delta f_p}^2$ for FM was calculated as follows:

$$\sigma_{\Delta f_p}^2 = \left(\frac{\partial \Delta f_p}{\partial \beta} \right)^2 \sigma_\beta^2 = f_m^2 \sigma_\beta^2 \quad (13)$$

The uncertainty of the attenuator could not be analytically propagated to the uncertainty of the FM and PM modulation indices because of the fitting process. Thus, Monte Carlo simulations were used to account for the uncertainty of the attenuator. The Monte Carlo simulations were performed separately for the reading and the calibration uncertainty of the attenuator because of their rectangular and normal distribution, respectively. The results are summarized in Tables 3 and 4. In addition, the impedance mismatch and repeatability uncertainties were accounted for in the same way as was done for AM. As shown in Fig. 2, the uncertainty of the impedance mismatch was evaluated by the insertion of the impedance tuner between the spectrum analyzer and the attenuator and the adjustment of the state of the tuner. The expanded uncertainty combining the above-described fitting uncertainty, attenuator uncertainty, impedance mismatch uncertainty, and repeatability was observed at 88.8 Hz to 571.4 Hz for FM ($k = 2$) and 0.88 mrad to 5.70 mrad ($k = 2$) for PM.

VI. CONCLUSION

We proposed an accurate method for measuring the analog modulation index. A calibrated attenuator was used to reduce measurement uncertainty and to ensure traceability. In the case of AM, the modulation index was measured by the power ratio of the first sideband and the carrier, as in the conventional method, and the measurement uncertainty was derived analytically. The 50% AM modulation index exhibited an expanded uncertainty ($k = 2$) of 0.372%. For FM and PM, the modulation index was measured through the nonlinear fitting of the Bessel function of the first kind by using the measured sidebands. Unlike the case for the Bessel null technique, the arbitrary modulation index could be measured. The measurement uncertainty obtained by the Monte Carlo simulation was 88.8 Hz for 10 kHz FM modulation and 0.88 mrad for 0.1 rad PM modulation. It is expected that the measurement uncertainty of analog modulation can be further improved with the application of an attenuator with a higher resolution.

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APPENDIX

The signal v_{SA} received by the spectrum analyzer is calculated as follows [14].

$$v_{\text{DUT}} = \frac{v_{\text{SA}}}{h S_{21}} (1 - \Gamma_{\text{DUT}} S_{11} - \Gamma_{\text{SA}} S_{22} - \Gamma_{\text{DUT}} \Gamma_{\text{SA}} (S_{21} S_{12} - S_{11} S_{22})) \quad (14)$$

where v_{DUT} , Γ_{DUT} , Γ_{SA} , S_{ij} , and h are the output signal of the DUT, the reflection coefficient of the DUT, the reflection coefficient of the spectrum analyzer, S-parameters of the attenuator, and the frequency response of the spectrum analyzer, respectively. The frequency response of the sampler in most spectrum analyzers can be calibrated using [14–16]. Here, we assume $h = 1$ due to the narrow bandwidth of the analog modulation. In addition, when Γ_{DUT} , Γ_{SA} , S_{11} , and S_{22} are negligibly small, the above formula is simplified as below.

$$v_{\text{DUT}} = \frac{v_{\text{SA}}}{S_{21}}$$

Note that most apparatuses have a small reflection coefficient. However, if the reflection coefficient is not so small that it is negligible, 10 dB attenuators with low reflection coefficients or tuners can be used as pads to reduce the mismatch between the DUT, the step attenuator, and the spectrum analyzer. Moreover, the output signal of the DUT can be calculated using (14) to include the mismatch effect. The corrected difference of the carrier and sidebands, ΔP_{corr} , can be calculated at a dB scale as follows:

$$\begin{aligned} \Delta P_{\text{corr}} &= 20 \log_{10} \frac{V_{\text{DUT}}^{\text{SB}}}{V_{\text{DUT}}^{\text{C}}} \\ &= 20 \log_{10} \left(\frac{V_{\text{SA}}^{\text{SB}} S_{21}^{\text{C}}}{V_{\text{SA}}^{\text{C}} S_{21}^{\text{SB}}} \right) = \Delta P_{\text{SA}} - Att_{\text{set}} \end{aligned} \quad (15)$$

where superscripts SB and C represent the sidebands and carrier, respectively.

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